

## QUESTION 2015

### Group - A

#### (Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following:

i) The differential coefficient of  $x^6$  with respect to  $x^3$  is

a)  $2x^3$

b)  $2x$

c)  $2x^2$

d) 2

ii) The degree and order of the differential equation  $\left(\frac{d^2y}{dx^2}\right)^{\frac{2}{3}} - 3\frac{dy}{dx} = 4$  are

a) degree =  $\frac{2}{3}$ , order = 2

b) degree = 2, order = 2

c) degree = 2, order = 1

d) degree = 3, order = 2

iii) The series  $1 - 1 + 1 - 1 + \dots$  is

a) convergent with sum 0

b) convergent with sum 1

c) divergent

d) oscillatory

iv) Let T be a linear transformation from  $R^2$  to  $R^3$  defined by

$$T(x, y) = (x + 2y, x - y, y).$$
 Then the image of (1, 2) is

a) (2, 1, -1)

b) (5, -1, 2)

c) (1, 1, 1)

d) (2, 2, 3)

v) In  $R^3$ , the vectors (1, 0, 1), (1, 1, 0) and (0, 1, 1) are

a) linearly dependent

b) linearly independent

c) both (A) and (B)

d) none of these

vi) If  $(5, 7) = a(1, 1) + b(1, 2)$  the values of a and b are respectively

a) 1, 2

b) 2, 3

c) 3, 2

d) 3, 3

vii) The series  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  is convergent if

- a)  $p \geq 1$                       b)  $p = 1$                       c)  $p \leq 1$                       d)  $p = 0$

Answer:  $p > 1$

viii)  $\frac{1}{(D-2)(D-3)} e^x$  is

- ✓ a)  $\frac{e^x}{2}$                       b)  $\frac{xe^{2x}}{2}$                       c)  $-\frac{xe^x}{6}$                       d)  $-\chi e^{3x}$

ix) If for a sequence  $\{u_n\}$ ,  $\lim_{n \rightarrow \infty} u_n = 0$  then

- a)  $\{u_n\}$  is convergent to 1                      b)  $\{u_n\}$  is divergent  
✓ c)  $\{u_n\}$  is convergent to 0                      d) none of these

x) If S and T be two subspaces of a vector space V, then which of the following is also a subspace of V?

- a)  $S \cup T$                       b)  $S-T$                       c)  $T-S$                       ✓ d)  $S \cap T$

xi) Integrating factor of  $ydx - xdy = y^2 \cos y dy$  is

- a)  $\frac{1}{y^2}$                       b)  $y$                       ✓ c)  $\frac{1}{y}$                       d) 1

xii) Leibnitz's test is applied to

- a) a constant series                      b) a series of positive terms  
✓ c) an alternating series                      d) a series of negative terms

xiii) Let T be a linear transformation from  $R^2$  to  $R^3$  defined by  $T(x, y) = (x + y, 0, 0)$ . Then rank of T is

- a) 3                      b) 2                      ✓ c) 1                      d) 0

### GROUP - B

(Short Answer Type Questions)

2. Solve any two of the following:

- a)  $y = px + \frac{a}{p}$                       b)  $(D^2 - 4)y = e^{2x} + e^{-4x}$                       c)  $(D^2 + 9)y = \cos 3x$

See Topic: DIFFERENTIAL EQUATIONS, Long Answer Type Question No. 2(ii), (iv) & (v).

3. Test the convergence of the series

$$x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \dots X > 0.$$

See Topic: SERIES, Short Answer Type Question No. 6.

4. Let  $S = \{(x, y, z) \mid x + y + z = 0, x, y, z \in R^3\}$ . Prove that S is a subspace of  $R^3$ . Find the dimension of S.

See Topic: LINEAR ALGEBRA, Long Answer Type Question No. 16.

5. Find the representative matrix of the linear transformation T :

$$R^3 \longrightarrow R^3 \text{ defined by}$$

$$T(x, y, z) = (3x + z, -2x + y, -x + 2y + 4z)$$

See Topic: LINEAR ALGEBRA, Short Answer Type Question No. 19.

6. Define monotone sequence. When is a monotone sequence convergent? Is the following sequence  $\{u_n\}$  convergent?

$$u_n = \frac{3n+1}{n+2}$$

See Topic: SEQUENCE, Short Answer Type Question No. 2.

### GROUP - C

(Long Answer Type Questions)

7. a) Verify whether the differential equation  $e^y dx + (xe^y + 2y)dy = 0$  is exact.

b) Solve :  $x \frac{dy}{dx} - 2y = xy^4$

c) Find the general and singular solutions of  $y = px - p^2$

a) & b) See Topic: DIFFERENTIAL EQUATIONS, Long Answer Type Question No. 9(b) & (c).

c) See Topic: DIFFERENTIAL EQUATIONS, Short Answer Type Question No. 6.

8. a) Discuss the convergency of the sequence  $\left\{ \frac{1}{n} \sin \frac{n\pi}{2} \right\}$

b) Let  $S = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a+b=0, a, b, c, d \in R \right\}$ . Find a basis and dimension of S.

c) Show that  $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$  is a divergent series

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- a) See Topic: SEQUENCE, Short Answer Type Question No. 1(c).  
b) See Topic: LINEAR ALGEBRA, Long Answer Type Question No. 2.  
c) See Topic: SERIES, Short Answer Type Question No. 14.

9. a) Solve :  $x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} - 5y = \sin \log x$

b) If  $\{\alpha, \beta, \gamma\}$  is basis of a real vector space  $V$ , show that  $\{\alpha + \beta, \beta + \gamma, \gamma + \alpha\}$  is also a basis of  $V$ .

c) Determine the linear mapping  $T : R^3 \longrightarrow R^3$  which maps the basis vectors  $(0, 1, 1), (1, 0, 1), (1, 1, 0)$  of  $R^3$  to the vectors  $(1, 2, 1), (1, 1, 2), (2, 1, 1)$  respectively, find  $\dim(\ker T)$ .

- a) See Topic: DIFFERENTIAL EQUATIONS, Short Answer Type Question No. 4.  
b) See Topic: LINEAR ALGEBRA, Long Answer Type Question No. 7(b).  
c) See Topic: LINEAR ALGEBRA, Short Answer Type Question No. 4.

10. a) State D' Alembert's ratio test. Test the convergence of the series  $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$

b) Show that the series  $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$  converges conditionally

c) Show that the sequence  $\sqrt{2}, \sqrt{2 + \sqrt{2}}, \sqrt{2 + \sqrt{2 + \sqrt{2}}}, \dots$  converges to 2.

- a) See Topic: SERIES, Long Answer Type Question No. 2(i) & Short Answer Type Question No. 1.  
b) See Topic: SERIES, Short Answer Type Question No. 2.  
c) See Topic: SEQUENCE, Long Answer Type Question No. 5.

11. a) Find the differential equation of all circles touching the axis of  $x$  at the origin.

b) Show that the vectors  $(1, -2, 3), (2, 3, 1)$  and  $(-1, 3, 2)$  form a basis of  $R^3$

c) Give an example to show that union of two sub spaces need not be a sub space of  $v$ .

a) See Topic: DIFFERENTIAL EQUATIONS, Long Answer Type Question No. 17.

b) See Topic: LINEAR ALGEBRA, Long Answer Type Question No. 6(b).

c) See Topic: LINEAR ALGEBRA, Long Answer Type Question No. 4(b).